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Reg. No._____ Name:_____

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY
THIRD SEMESTER B.TECH DEGREE EXAMINATION, JULY 2017

Course Code: **MA 201**
Course Name: **LINEAR ALGEBRA AND COMPLEX ANALYSIS.**

Max. Marks :100 Duration: 3 hours

PART A

Answer any two questions.

1. (a) Does the limit $Lim_{z\to0}\frac{z}{z}$ exit? If yes find the value. If no, explain why? **(8)**

    (b) If $f(z) = u + iv$ is analytic, prove that u = constant and v = constant are families of curves cutting orthogonally **(7)**

2. (a) Find the image of the semi-circle $y = +\sqrt{4-x^2}$ under the transformation $w = z^2$ **(7)**

    (b) Find the image of the half-plane $Re(z) \geq 2$ under the map $w = iz$ **(8)**

3. (a) Find the points, if any, in complex plane where the function $f(z) = 2x^2 + y + i(y^2 - x)$ is

    (i) differentiable (ii) analytic. **(8)**

    (b) Prove that the function $u(x,y) = x^3 - 3xy^2 - 5y$ is harmonic everywhere. Also find the harmonic conjugate of u. **(7)**

PART B

Answer any two questions.

4. (a) Evaluate $\int_C \bar{z}\,dz$ where C is given by $x = 3t, y = t^2, -1 \leq t \leq 4.$ **(8)**

    (b) Show that $\int_C (2+z)^2 dz = -\frac{i}{3}$ where C is any path connecting the points -2 and -2 + i **(7)**

5. (a) Evaluate $\int_C \frac{5z+7}{z^2+2z-3} dz$ where C is the circle $|z - 2| = 2.$ **(8)**

    (b) Find the Laurent's series expansion of $\frac{1}{z-z^3}$ in $1 < |z + 1| < 2.$ **(7)**

6. (a) Use Cauchy's integral formula to evaluate $\int_C \frac{z+1}{z^4+2iz^3} dz$ where C is $|z| = 1.$ **(8)**

    (b) Using Contour integration, evaluate $\int_{-\infty}^{\infty} \frac{x^2-x+2}{x^4+10\ x^2+9} dx$ **(7)**

PART C

Answer any two questions.

7. (a) Using Gauss elimination method, find the solution of the system of equations
$x + 2y - z = 3, 3x - y + 2z = 1, 2x - 2y + 3z = 2$ and $x - y + z = -1$ **(7)**
(b) Find the values of μ for which the system of equations $x + y + z = 1$, $x + 2y + 3z = \mu$ and $x + 5y + 9z = \mu^2$ will be consistent. For each value of μ obtained, find the solution of the system.   **(7)**
(c) Prove that the vectors (2,3,0). (1,2,0) and (8,13,0) are linearly dependent in R^3.
   (6)

8. (a) Find the rank of the matrix $A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -1 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$ **(7)**

(b) Find the eigen values and eigen vectors of the matrix $\begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$ **(7)**

(c) Write the canonical form of the quadratic form $Q(x, y, z) = 3x^2 + 5y^2 + 3z^2 - 2xy + 2xz - 2yz$ and hence show that $Q(x, y, z) > 0$ for all non-zero values of x, y, z.   **(6)**

9. (a) Diagonalize the matrix $A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$ and hence find A^4.   **(7)**

(b) If 2 is an eigen value of $\begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$, without using its characteristic equation, find the other eigen values. Also find the eigen values of $A^3, A^T, A^{-1}, 5A, A - 3I$ and $adj\ A$.   **(7)**

(c) Show that $17x^2 - 30xy + 17y^2 = 128$ represents an ellipse. Also find the equations of the major and minor axes of the ellipse in terms of x and y.   **(6)**

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